VOLUME ESTIMATION FOR HIGHWAY CURVES USING END-AREA RULE

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Abstract - The volume calculation of road sub surface materials in highways is important in project cost estimation prior to the construction, and in investigations after the construction. Usually it is done with the linear measurements and a level line as the field measurements, and the end-area-rule is the most widely used equation in the calculation. This theory provides the volume between two consecutive sections which should be parallel to each other. Curve shape is often neglected in most of the construction projects due to the difficulty of conducting an advanced control network survey. Therefore, calculated material volume of a road bend might drastically deviate from the original material volume. This study was carried out to develop a physical model to determine the volume between two cross sections which are not parallel. Initially the scope of the study was limited to a circular curve with uniform variation of cross section. The volumes between two cross sections for different angles were measured using the mathematical and physical models in order find the correlation between them. It was found that the end-area-rule is valid for circular curves and the developed physical model provides an acceptable estimation of volume. The outcome of the research would assist construction professionals to identify the error of the previously used method and to overcome the predicament.

Keywords: Circular curves, volume calculation, end-area-rule

I. INTRODUCTION

The road construction projects become very important development activity for a country as it is directly related to the economy through factors such as transport, tourism and security. Volume calculation of road sub surface materials is the most complex event in the cost estimation as it is varying according to the terrain. This volume estimation is required to estimate the cost of the total project at the planning stage and also used in investigations on factors such as material usage and workmanship conducted after the construction.

Calculating the volume with cross sections is the most widely used and the most convenient method used in the analysis since it can be conducted with linear measurements and a level line run through the centre line and along cross sections which are perpendicular to the road centre line. The cross sections are usually generated from these measurements and the sectional areas measured from these sections are used to calculate the volume. The equation called end-area-rule is usually used to calculate the volume between two such cross sections which are at a known distance apart. This theory has been derived for two parallel sections with a linear variation of sectional area over the distance between them. Even though the rule is valid under these conditions, it is widely used for skewed sections in volume calculation of road materials. This negligence of parallelism of sections occurs due to the curves in highways leads to errors in the volume calculation. The angle between the sections can not be measured with linear measurements and it can be determined by conducting an advanced control survey.

This study was conducted to analyse the effect of skew angle between the cross sections on the volume calculation when end area rule is used for such situation. The actual area calculated using a mathematical model using integration for curve with defined shape. A physical model was developed to measure the volume for sections with varying angles of cross sections. The result was correlated to determine the relationship between these models and also used to determine the modification for the end area rule to be applied when it is used for curved highways.

II. LITERATURE REVIEW

The material volume is one of the most important objectives in horizontal optimization, so most researches firstly focused on the measuring material volume to control the cost. End area rule is the most commonly used technique in volume calculation for road construction projects. It is used to find the volume between two parallel sections assuming the sections vary linearly. The volume is estimated by multiplying the average area by the distance between the sections.

There are various researches have been done regarding the volume estimation. As stated by Easa (1992), existing method of average volume is only an approximation and it is only applicable for level terrains. An exact method is not available for fluctuating profiles as the end area rule assumes a linear variation. The model introduced by Easa was based on triple integration, assuming that the ground cross slope is constant between stations. The application results indicate that the volume of the average-volume method is deviating greatly from the exact volume and that the mathematical model is also reasonably accurate when the grounds cross slope changes moderately.

According to Easa (2003), the traditional model for estimating earthwork volumes of curved roadways (flat horizontal curves) is suitable only for level terrains. For moderately fluctuating terrains, a mathematical model has been developed. This model, however, assumes that the longitudinal ground profile between successive stations is linear and the ground cross slope is constant. The mathematical model is not accurate for greatly fluctuating profiles, such as those in hilly and mountainous terrains.
Cheng (2005) conducted a study to solve the inaccuracy problem caused by average end-area method and prismoidal method used for the calculation of roadway earthwork volume. Further it was presented in a complete 3-dimensional algorithm of roadway earthwork volume as well as its executable computer program. The algorithm benefits from the re-triangulation technique of constrained Delaunay triangulation (CDT), which can yield a true volume value theoretically.

Cheng and Jiang (2013) reconfirmed the feasibility of average-end-area method for earthwork volume and the analysis of difference of accuracy between 3D method and average-end-area method. It shows that the critical value of interval distance between two consecutive cross sections is 30m for average-end-area method. It is also concluded that the 3D method could be easily used in practice with the CAD software. Meanwhile, average-end-area method with less than the critical interval distance between two consecutive cross sections can guarantee the earthwork calculation accuracy.

Hu (et. al., 2015) proposed that assessment of slope excavation can be performed with laser scanned data together with the quality control indices such as average gradient, slope toe elevation, over break and under break, cross-sectional quality assessment and holistic quality assessment methods. An algorithm was also presented to calculate the excavated volume with laser-scanned data. It is also stated that time consumption can also be deducted from 70% by using laser scanning technology for excavation quality assessment than traditional method. Khalil (2015) stated that the average end area method is tedious and time consuming. Volume of terrains that do not have regular geometric structure can be obtained more accurately by using 3D models of surfaces with respect to developing technology such as GIS. The gridding method and point distribution are important factors in modelling earth surfaces used for volume estimation. The results show that for gentle slope surface, Triangular Irregular Network (TIN) and all interpolation techniques gave results very close to the exact except Kriging and Trend interpolation. Kriging interpolation gave the best results for steep slope terrain.

The above studies demonstrate that the end area rule can be use for the volume estimation together with proper adjustments in order to achieve higher accuracy for curved roads.

III. METHODOLOGY

C. Mathematical Model

Initially a circular curve was selected in order to develop the physical model and compare the volume measured by different methods. A circular section of radius R with uniform width was considered as in Figure 1 and the variation of thicknesses were selected such that they vary linearly along the outer and inner edges. This factor assured the major assumption of end area rule that is the variation of sectional areas to be linear. Then a small section at α angle with small $d\alpha$ was considered as per the figure. The heights of corners were calculated by interpolation and the linear integration was used to calculate the volume of the section mathematically.

Following notations were used in the mathematical model.

R	- Radius of the curve
W	- Width of the section
θ	- Angle of the curve
α	- distance to the considered small section
dα	- small angle
hi	- corner height of the section
h’	- height of the small section at outer edge
h”	- height of the small section at inner edge

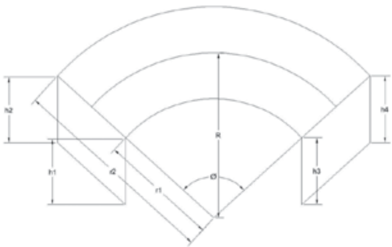

Figure 1. The mathematical model

The heights h’ and h” calculated using linear interpolation considering the linear variation along the edges.

$$h' = h_1 + \left(\frac{h_4 - h_1}{\theta}\right)\alpha$$

$$h'' = h_2 + \left(\frac{h_3 - h_2}{\theta}\right)\alpha$$

The volume calculated by integration = ∫dv

$$= \int_0^\theta \left(\frac{h' + h''}{2}\right) WR.d\alpha$$

$$= \int_0^\theta \left[\frac{h_1 + \left(\frac{h_4 - h_1}{\theta}\right)\alpha + h_2 + \left(\frac{h_3 - h_2}{\theta}\right)\alpha}{2}\right] wR.d\alpha$$

$$= \int_0^\theta [\left(\frac{h_1 + h_2}{2}\right) WR + \frac{\alpha}{\theta}(h_4 - h_1 + h_3 - h_2)]WR.d\alpha$$

$$= \left(\frac{h_1 + h_2}{2}\right) WR \int_0^\theta d\alpha + \frac{WR}{2}\int_0^\theta (h_4 - h_1 + h_3 - h_2)\alpha.d\alpha$$

$$= \left(\frac{h_1 + h_2}{2}\right) WR\theta + \frac{WR}{2\theta}(h_4 - h_1 + h_3 - h_2)\left[\frac{\theta^2}{2}\right]$$

$$= \frac{WR\theta}{2}\left[h_1 + h_2 + \left(\frac{h_4 - h_1 + h_3 - h_2}{2}\right)\right]$$

$$= \frac{WR\theta}{4}[h_4 + h_1 + h_3 + h_2]$$

$$= \frac{h_1 WR\theta}{4} + \frac{h_2 WR\theta}{4} + \frac{h_3 WR\theta}{4} + \frac{h_4 WR\theta}{4}$$

$$= \left(\frac{h_1 + h_2}{2}\right)\frac{WR\theta}{2} + \left(\frac{h_3 + h_4}{2}\right)\frac{WR\theta}{2}$$

Since; $A_1 = \left(\frac{h_1 + h_2}{2}\right) W$ & $A_2 = \left(\frac{h_3 + h_4}{2}\right) W$

$$= \left(\frac{A_1 + A_2}{2}\right) R\theta$$

= Average area x Distance between the sections

D. Physical Model

The physical model is developed by carving a circular groove with constant width and a thickness varying linearly along each edge. The radial lines at various angles were marked and a stop board was used to limit the volume at different angles. The height of the groove at each radial line was measured and ensured that they vary linearly. The space was filled by sand sieved from number 200 sieve with a fluviation height less than 1 cm and the this volume of sand is measured with a measuring cylinder. This procedure was repeated for different angles by changing the stop board.

Figure 2. The physical model

E.Correlation of Volumes

A calibration was needed for the physical model in order to calculate the actual volume from the sand volume used to fill the space. Cubical grooves with known dimensions were carved in the same base as the physical model and those spaces were filled with the sand in the same procedure. These volumes were measured with the measuring cylinder and they were compared with corresponding actual volumes calculated by dimensions to find the correlation between the actual volume and the sand volume.

IV. RESULTS

F. Correlation of Volumes

The sand volumes of cubical grooves and the volumes calculated by dimensions were given in Table 1.

Table 1. Sand volumes in calibration

Dimensions /cm	Volume (cm3)	Sand volume/ ml
3.5 x 3.5 x 2.5	30.625	34.0
3.5 x 3.5 x 3.5	42.875	48.0
3.5 x 3.5 x 4.5	55.125	62.0
2.5 x 2.5 x2.5	15.625	17.5
2.5 x 2.5 x 3.5	21.875	24.5
2.5 x 2.5 x 4.5	28.125	31.0

As per the Figure 3 it is found that the variation of sand volume and the actual volume is linear and the correlation factor is 0.893 where sand volume is taken in millilitres while actual volume is in cubic centimetres.

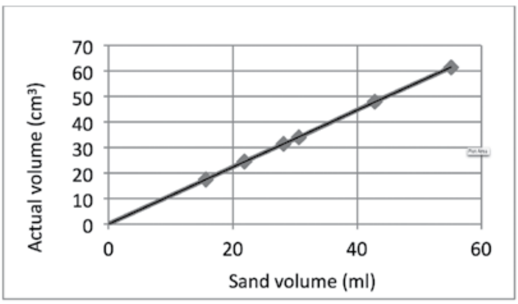

Figure 3. The variation of sand volume with actual volume

G. Volume Measurements

The measurements obtained to calculate the volumes for physical model and the mathematical model are given in Table 2. The skew angle was controlled by stop board and the sand volume was measured to obtain the volume of physical model and heights of corners were measured for the volume of mathematical model. The initial corner heights (h1 and h2) were 2.2 cm and 1.2 cm and the radius and width were 20 cm and 10 cm respectively.

Table 2. Measurements for volume calculation

Skew Angle	Corner heights (mm)	Sand volume/ ml
00o	2.2, 1.2	-
15o	2.6, 1.8	142.0
30o	2.9, 2.4	318.0
45o	3.3, 3.0	533.0
60o	3.6, 3.6	775.0
75o	4.0, 4.2	1065.0

The calculated volumes for both physical and mathematical models are given in the Table 3 and their variations were illustrated in Figure 4.

Table 3. Calculated volumes for different methods

Skew Angle	Calculated Volume / (cm3)	
	Physical Model	Mathematical Model
15o	127.63	126.81
30o	284.71	283.97
45o	476.15	475.97
60o	693.79	692.08
75o	949.02	951.05

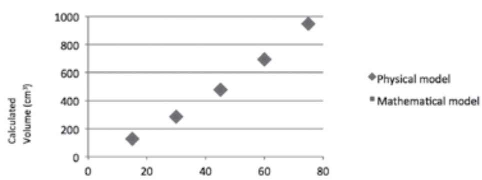

Figure 4. The variation of calculated volumes

V.CONCLUSION & DISCUSSION

The study confirmed that the end area rule can be directly applied to circular curves neglecting the criterion that the sectional areas should be parallel. The physical model developed by this study can be used to estimate the volume between two cross sections by considering the end area rule. It is recommended to extend this research towards a curve with non circular profile.

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